

Research Paper

Open Access

Smooth Transition Exponential Smoothing Method in Value at Risk Forecasting

Cao Yangfan^{1*}, Choo Wei Chong², Bolaji Tunde Matemilola³

¹School of Business, Sunway University, Sunway City, Selangor, Malaysia;

²Department of Management and Marketing, Faculty of Economics and Management, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia;

³Department of Accounting and Finance, Faculty of Economics and Management, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia

Publication Details: Received 9/5/2024; Revised 4/6/2024; Accepted: 22/8/2024

ABSTRACT

As a representative adaptive model, smooth transition exponential smoothing (STES) has demonstrated its robustness by enhancing the precision of volatility forecasting in various domains. However, limited attention has been devoted to its application in Value at Risk (VaR) forecasting. This study aims to address this gap by introducing STES with diverse selections of transition variables for daily VaR forecasting and assessing its suitability as a VaR model. Additionally, the study compares adaptive methods and the well-established generalised autoregressive conditional heteroskedasticity (GARCH) family, specifically in the context of the COVID-19 pandemic, for VaR forecasting. The analysis includes six daily international stock indexes from North America, Europe, and Asia, offering a solid foundation for comparing the performance of different forecasting models. This is especially important because a reliable model proven in developed stock markets can offer valuable insights for developing Asian markets, aiding in better risk management and enhancing financial stability during rapid economic growth. The post-sample results reveal that the STES method consistently outperforms other models, particularly at lower risk levels when employing prior shocks as a transition variable. Besides, the findings show that using multiple transition variables in STES estimation results in underestimating true risk, potentially compromising the model's robustness.

Keywords: Value at risk (VaR); smooth transition exponential smoothing (STES); adaptive methods; COVID-19 pandemic; daily VaR forecasting; GARCH model

Introduction

The global economic downturn triggered by the onset of the COVID-19 pandemic has brought about heightened concerns regarding risk management. Particularly, the pandemic's profound impact and extreme spillovers have significantly contributed to amplified instances of extreme losses (Cao, 2022). During this period, the concept of Value-at-Risk (VaR), a vital tool for risk measurement, has garnered increased attention and discussion in both academic and industrial circles. VaR plays a pivotal role in risk management across various sectors, having gained widespread acceptance as the primary risk metric, notably endorsed by the Basel Committee on Banking Supervision (BCBS). In essence, VaR represents a straightforward statistical concept and a practical instrument for gauging risk. It quantifies the

maximum potential loss within a specified time and with a defined level of confidence. Notably, the pioneering work of J.P. Morgan's risk management team, who developed Risk Metrics Models using the VaR approach (Longestae & Spencer, 1996), spurred growing interest among researchers in the theoretical development and practical application of VaR forecasting. In practice, market risk constitutes a pivotal component in the calculation of regulatory capital, as stipulated in Pillar I of Basel II, prompting the BCBS to formulate comprehensive supervisory and administrative guidelines and requirements grounded in VaR principles (Jiménez-Martín, McAleer, & Pérez-Amaral, 2009). Consequently, due to heightened financial trading activities in recent years, market risk has assumed heightened significance, drawing substantial attention from both managers and regulatory authorities.

Expected Shortfall (ES) has gained increasing attention recently, particularly following the proposal in the Basel III Accord to encourage its implementation (Chang, Jiménez-Martín, Maasoumi, McAleer, & Pérez-Amaral, 2019). This recommendation became particularly relevant during the COVID-19 recession, as financial institutions were required to maintain higher capital reserves. However, it's worth noting that excessive regulation has been identified as a potential contributor to the financial crises. For instance, the Basel II Accord received criticism for mandating banks to increase capital during times of rising risks, which, paradoxically, could lead to reduced lending precisely when capital is in short supply, potentially exacerbating financial crises.

Given these considerations, our study primarily focused on VaR forecasting. This study believes that VaR should take precedence, serving as the fundamental basis for risk measurement, whether in periods of financial crisis or stability (Hoga, 2021). Moreover, achieving precise control or forecasting of financial risk is of paramount importance. Such precision can empower decision-makers, including managers and regulators, to make informed choices regarding risk management (Meng & Taylor, 2020). Despite the conceptual clarity of VaR, it's important to acknowledge that obtaining a precise measurement of conditional VaR is statistically challenging (Bali, Mo, & Tang, 2008). Additionally, Abad and Benito (2013) have emphasized the substantial impact of volatility models on the accuracy of VaR forecasting. In essence, the accuracy of VaR forecasting can be significantly enhanced through the use of precise volatility models. Even though the analysis includes several international stock indexes from North America, Europe, and Asia, a proven reliable model in the developed stock market could provide valuable insights for developing markets (Kose, Prasad, Rogoff, & Wei, 2009). This study is particularly relevant to the rapidly growing Asian market. Improved risk forecasting can significantly benefit financial institutions and enterprises in this region by offering more accurate assessments of potential losses. As a result, better capital allocation and strategic planning are possible when addressing market risk, which eventually helps to build a stronger financial system in Asia. Implementing advanced risk management strategies like STES can promote sustainable economic growth and enhance the region's financial stability.

The smooth transition exponential "moot'Ing (STES) method proposed by Taylor (2004) was used for volatility forecasting in financial returns and it was proven as a robust and precise model in volatility forecasting (Gooi, Choo, Nassir, & Immng, 2018). Some advantages of STES may contribute to successful VaR forecasting. Firstly, it enables a logistic function of a user-specified transition variable to be employed as an adaptive, time-varying smoothing parameter, which accelerates VaR change simultaneously and is more accurate (Polanski & Stoja, 2010). Secondly, the asymmetry effect in the volatility of financial asset" return, namely the "leverage effect", plays a vital role in VaR forecasting (Su, 2014). In a previous

STES study, the leverage effect has been modelled by using the sign of prior shocks as a transition variable (Taylor, 2004). This asymmetry is characterized by the propensity for periods of greater volatility to follow periods of negative returns than by periods of positive returns of similar size. Therefore, the magnitude of the prior shocks has also been introduced as a transition variable, to enable more adaptable modelling of the dynamics of the conditional variance.

Regarding parameter estimation, Taylor (2004) recommended minimizing the sum of squared deviations between realized volatility and predicted volatility to derive parameters for the method. Given the cost of high-frequency data for volatility calculations, Liu, Taylor, and Choo (2020) proposed the use of STES for daily volatility forecasting based solely on daily returns, without relying on realized volatility. Additionally, STES exhibits robust characteristics in the presence of outliers when employed for time series level prediction (Liu et al., 2020; Taylor, 2004). Furthermore, several studies have applied the STES method across diverse industries and consistently found it to outperform models from the widely recognized GARCH family (Gooi et al., 2018; Kin, Chong, Nassir, Habibullah, & Yusop, 2021; Ung, Choo, Sambasivan, & Nassir, 2014; Zhang, Choo, Ho, & Wan, 2022).

This study focuses on enhancing the accuracy of VaR forecasting by incorporating precise volatility models, specifically the STES models. While the STES method has demonstrated effectiveness in volatility prediction (Gooi, Choo, Nassir, & Immng, 2018; Taylor, 2004), its utilization in VaR forecasting remains relatively unexplored. The research also conducts a comparison, taking into account VaR-specific characteristics such as volatility clustering, asymmetry of effects, and long memory. Additionally, our investigation focuses on whether the STES model can yield accurate VaR forecasting during the COVID-19 period. This new comparison of STES and long memory models such as ARFIMA and FIGARCH models has not been implemented in previous research.

The selection of transition variables is of essential significance to the success of STES. It has been proposed that trading volume can be a useful predictor of volatility. For instance, GARCH models developed by Lamoureux and Lastrapes (1990), Brooks (1998), and Donaldson and Kamstra (2005) incorporate trading volume as a regressor. Despite being tested as a STES transition variable, trading volume did not provide accurate volatility forecasting (Liu et al., 2020), and it has not been verified in VaR forecasting. In this study, the utility of trading volume will be assessed as a STES transition variable.

In Section 2, related literature on GARCH models (i.e., the conventional GARCH models, asymmetric type GARCH models, adaptive type GARCH models, and long-memory Models) and their application in VaR forecasting are reviewed. In Section 3, the STES model is introduced along with a description of the model construction. In Section 4, six international stock indices are used to evaluate the performance of the new methods when compared with several alternative models. Section 5 summarizes and concludes the paper.

Literature Review

The current VaR forecast methods can be classified into three types: 1) parametric, 2) nonparametric, and 3) semiparametric. Nonparametric methods don't make any assumptions about the distribution of returns or the parametrization of conditional volatility. In parametric models, conditional volatility is parameterized, and the return distribution is a prerequisite. Semiparametric methods include those based on the extreme value approach, such as McNeil

and Frey's (2000) model, which employs the peaks over threshold extreme value theory (EVT) method to standardize residuals for volatility using GARCH conditional volatility forecasting. Besides, quantile regression-based methods, such as the conditional autoregressive value at risk (CAViaR) models of Engle and Manganelli (2004), are also categorized as semiparametric VaR methods.

The VaR forecasting derived from STES involves an initial step of volatility forecasting, leading to the selection of comparison models based on parametric methods. This choice is made considering the presence of specific features in volatility specification, such as volatility pooling, the leverage effect, and long memory. Previous studies have introduced parametric models tailored to capture these distinctive features. Consequently, we include these clustering, leverage effect, and long-memory models in our study for comparison against our STES models. Besides, the most common distributional assumption will be adopted for the parametric methods in this study which the financial returns follow the student-t distribution. The following formula can be used to determine the VaR at different probability values under the student-t distribution assumption (Alexander, 2009):

$$VaR_t = \hat{\mu}_t + q_\alpha \hat{\sigma}_t \quad (1)$$

$\hat{\mu}_t$ stands for the conditional mean, $\hat{\sigma}_t$ for the conditional standard deviation, and q_α for the critical t-value at $\alpha\%$ for the student-t distribution. In the rest of the contents, this study will review the characteristics of numerous conditional variance models and the conditional mean model.

Conditional Mean Model

Assume that P_t represents the price of the asset at time t and $r_t = \log(P_t) - \log(P_{t-1})$ represents the asset return from time $t - 1$ to time t , in which the independent random variables $r_1, r_2, r_3, \dots, r_t$, are identically distributed. The models will compare rely on r_t performing the following steps (Engle & Patton, 2001):

$$r_t = \mu_t + \sigma_t \epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } F(\cdot) \quad (2)$$

Where F is the cumulative distribution function of ϵ , μ_t and σ_t are conditional mean and conditional volatility. For a parametric model, a complete model should describe the dependence of μ_t and σ_t on the historical data as well as the distribution F . According to Angelidis, Benos, and Degiannakis (2004), the mean process specification has no significant impact on VaR forecasting. Therefore, throughout the paper, the conditional mean μ_t is modelled for construction VaR by a linear autoregressive moving average ARMA (1, 1) model:

$$\hat{\mu}_t = \alpha_0 + \beta_1 r_{t-1} + \epsilon_t + \theta \epsilon_{t-1} \quad (3)$$

Here, ϵ_t is regarded as the error term, which is determined as follows: $\epsilon_t = r_t - \mu_t$. This term is commonly referred to as the price "news" or "shock."

Conditional Variance Model

Traditional GARCH

ARCH models, which were first proposed by Engle (1982), represent conditional variance as a linear function of lagged squared error terms. The GARCH models, which Bollerslev (1986) created as an extension of the ARCH models, provide for a sparser representation in several practices. GARCH models utilize lagged squared error terms and lagged conditional variance terms to express conditional variance as a linear function. For instance, this is demonstrated through the GARCH (1, 1) model presented below:

$$\hat{\sigma}_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (4)$$

Where $\omega > 0$, α_1 and β_1 both are non-negative coefficients so the predictive variance is positive and stationary.

Engle and Bollerslev (1986) created the IGARCH model to study volatility persistency, which is represented by:

$$\hat{\sigma}_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + (1 - \alpha_1) \sigma_{t-1}^2 \quad (5)$$

The present news shock's effect on volatility will continue to linger and affect future variations (Choudhry, 1995). when the unconditional variance undergoes a structural shift, this could result in inaccurate predictions.

Asymmetric Models

Studies of financial stock returns frequently reveal that periods of greater volatility follow periods of negative returns than periods of positive returns of equivalent size. This asymmetry can be explained by the fact that both positive and negative shocks affect a company's leverage differently, which in turn affects how volatile it is (Black, 1976). By implementing more adaptive volatility treatments that take into consideration the asymmetric volatility caused by positive and negative shocks, this leverage effect has been addressed. Since the conditional variance is not modelled as a linear function of lagged squared error and lagged variance, these models are modelled as non-linear. The exponential GARCH (EGARCH) model (Nelson, 1991) and the Glosten-Jagannathan-Runkle GARCH (GJRARCH) model (Glosten, Jagannathan, & Runkle, 1993) are two straightforward kinds of models that reflect the asymmetric nature of returns. The conditional variance EGARCH (1, 1) model's specification is as follows:

$$\ln(\hat{\sigma}_t^2) = \omega + \alpha_1 \frac{|\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2) \quad (6)$$

When ε_{t-1} is positive, it increases $\alpha_1(1 + \gamma_1) |\varepsilon_{t-1}/\sigma_{t-1}|$ to the log volatility, while when ε_{t-1} is negative, it adds $\alpha_1(1 - \gamma_1) |\varepsilon_{t-1}/\sigma_{t-1}|$.

Another general asymmetric GJRARCH (1, 1) is specified as:

$$\hat{\sigma}_t^2 = \omega + (\alpha_1 + \gamma_1 N_{t-1}) \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (7)$$

Where N_{t-1} is an indicator function, that is,

$$N_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{if } \varepsilon_{t-1} \geq 0 \end{cases}$$

And α_1 , γ_1 and β_1 are non-negative coefficients satisfying conditions like those of GARCH models.

Adaptive Models

There have been numerous studies applying smooth transition models in GARCH modelling. These models are characterized by the modelling of at least one parameter as a continuous function of a transition variable. In 1997, Hagerud introduced the logistic smooth transition (LSTGARCH) model, designed to achieve a smooth transition between the α and γ coefficients of the lagged squared error factors found in the GJR GARCH model. An LSTGARCH (1, 1) model is represented as follows:

$$\hat{\sigma}_t^2 = \omega + (1 - F(\varepsilon_{t-1}))\alpha_1\varepsilon_{t-1}^2 + F(\varepsilon_{t-1})\gamma_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (8)$$

Where $F(\varepsilon_{t-1})$ is a logistic function, that is:

$$F(\varepsilon_{t-1}) = \frac{1}{1 + \exp(-\theta\varepsilon_{t-1})} \quad (9)$$

The logistic function's range is from 0 to 1, and it varies depending on the variation of the transition variable ε_{t-1} . When $\theta > 0$, the logistic function is a ε_{t-1} function that monotonically increases. Thus, the effect of ε_{t-1} progressively change from α_1 to γ_1 when ε_{t-1} rises from a large negative number to a large positive number.

Additionally, the exponential smooth transition GARCH model (ESTGARCH) is suggested by Hagerud (1997). The logistic function in equation (8) of the ESTGARCH (1, 1) model is changed to the following exponential function, which has the same formula as the LSTGARCH (1, 1) model:

$$F(\varepsilon_{t-1}) = 1 - \exp(-\theta\varepsilon_{t-1}^2) \quad (10)$$

The conditional variance is dynamically varied depending on the size of the shock due to the exponential function. Because an exponential function enables $F(\varepsilon_{t-1})$ to vary between 0 and 1 as ε_{t-1}^2 varies between its extremes, it is employed instead of a logistic function.

Maximum Likelihood is used to estimate the parameters in the conditional mean model and conditional variance model forecasting. The log-likelihood function in this study assumes that the density of the error terms follows the student-t distribution.

Long-memory Models

The Autoregressive fractionally integrated moving average (ARFIMA) model, introduced by Granger and Joyeux in 1980, represents an advancement beyond the ARMA model, specifically designed to accurately capture the enduring memory effects within time series data. Unlike the ARIMA model, where deviations from the long-term mean exhibit an exponential decline, the ARFIMA model displays a hyperbolic slow decay, effectively

capturing the pronounced long-memory characteristics inherent in the time series. The ARFIMA model can be formally defined as:

$$\Phi_p(L)(1-L)^d(r_t - \mu) = \theta_q(L)\varepsilon_t \quad (11)$$

Where ε_t is assumed to be an i.i.d normal random variable with zero mean and variance σ^2 and L represents a lag operator, d is a differencing parameter, and $\Phi_p(L)$ and $\theta_q(L)$ are lag polynomials displayed below:

$$\Phi_p(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p$$

$$\theta_q(L) = 1 - \theta_1 L - \theta_2 L^2 - \dots - \theta_q L^q$$

$(1-L)^d$ is the differencing lag operator and can be rewritten as:

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \dots$$

To maintain stationarity in the ARFIMA process, the fractional integration parameter 'd' spans from -0.5 to 0.5. When 'd' falls within the range of -0.5 to 0, the ARFIMA process exhibits characteristics indicative of intermediate memory. Furthermore, when 'd' resides in the interval 0 to 0.5, the ARFIMA process transforms into a state characterized by mean reversion, long-term memory, and stationarity.

Based on prior studies, it has been observed that the autocorrelations of squared or absolute returns from diverse financial assets display characteristics suggestive of long-range dependence. These observations spurred the development of the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) model by Baillie, Bollerslev, and Mikkelsen in 1996. The primary objective behind creating the FIGARCH model was to construct a more flexible framework for modeling conditional variance processes, and capable of explaining and representing the temporal relationships inherent in the observed volatility of financial markets. This model not only aligns with the leptokurtic unconditional distribution of returns and the long-memory characteristics of conditional variances but also accommodates the temporal dependence of variance. The FIGARCH class (p, d, q) can be formally expressed as:

$$[1 - \alpha(L) - \beta(L)](1-L)^d \varepsilon_t^2 = \alpha_0 + [1 - \beta(L)]v_t \quad (12)$$

Where $\alpha(L)$ and $\beta(L)$ are lag operators the same as $\Phi_p(L)$ and $\theta_q(L)$ mentioned above, and $v_t = \varepsilon_t^2 - \sigma_t^2$. d means differencing parameter and usually ranges from 0 to 1 for revealing the long-memory effect.

Smooth Transition Exponential Smoothing in Var Forecasts *Characteristics of STES*

According to numerous authors, the time series' most significant present changing features may be captured by a time-varying smoothing value. An adaptive exponential smoothing was created by Taylor (2004) and is based on the smooth transition model. It was displayed that the STES generated a consistent and robust prediction. The method was formulated as:

$$\hat{\sigma}_t^2 = \alpha_{t-1} \varepsilon_{t-1}^2 + (1 - \alpha_{t-1}) \sigma_{t-1}^2 \quad (13)$$

Where α_{t-1} is a logistic function, that is:

$$\alpha_{t-1} = \frac{1}{1 + \exp(\beta + \gamma V_{t-1})}$$

The smoothing parameter, α_{t-1} , which ranges from 0 to 1, varies based on the dynamical transition variable, V_{t-1} , in which β and γ are constant parameters. Taylor (2004) proposed ε_{t-1} , $|\varepsilon_{t-1}|$, ε_{t-1}^2 individually and the combination of them as transition variables for STES in volatility forecasting. Besides, Liu et al. (2020) introduced two new transition variables LnVol_{t-1} and IndVol_{t-1} to the STES method. IndVol_{t-1} is an indicator consisting of the high/low volume, and LnVol_{t-1} represents a natural logarithm of lagged volume. The IndVol_{t-1} is defined as:

$$\text{IndVol}_{t-1} = \begin{cases} 1 & \text{if } \text{IndVol}_{t-1} \geq \frac{1}{n-1} \sum_{i=2}^n \text{Volume}_{t-i} \\ 0 & \text{if } \text{IndVol}_{t-1} < \frac{1}{n-1} \sum_{i=2}^n \text{Volume}_{t-i} \end{cases}$$

The parameters can be calculated by minimizing the sum of squared in-sample prediction errors. The minimization is specified as follows:

$$\min \sum_i (\varepsilon_i^2 - \hat{\sigma}_i^2)$$

In this study, the parameters optimization method will be implemented through R programming. Table 1 presents and details each transition variable utilized in the STES method for this study.

Table 1: Description of transition variable

Transition variables	Transition variables
E	ε_{t-1}
AE	$ \varepsilon_{t-1} $
SE	ε_{t-1}^2
E&AE	ε_{t-1} and $ \varepsilon_{t-1} $
E&SE	ε_{t-1} and ε_{t-1}^2
IndVol	High/low volume indicator
IndVol&AE	High/low volume indicator and $ \varepsilon_{t-1} $
IndVol&SE	High/low volume indicator and ε_{t-1}^2
LnVol	Log volume
LnVol&AE	Log volume and $ \varepsilon_{t-1} $
LnVol&SE	Log volume and ε_{t-1}^2

Empirical study

Data Description

A comparative analysis was conducted between the new VaR forecasting using the STES method and several GARCH models to assess its effectiveness. This analysis included stock indices from the world's six largest stock markets: United States (SPY 500). Canada

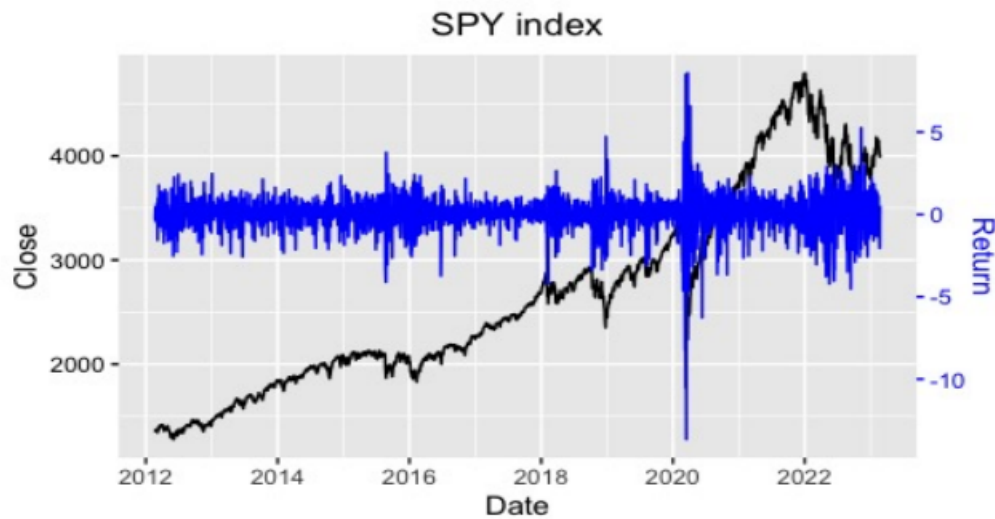
(GSPTSE), Hong Kong (HSI), Japan (NIKKEI), Amsterdam (AEX), France (FCHI). The stock indexes from North America, Asia, and Europe were chosen because of their substantial market capitalization, making them suitable samples to represent their respective continents. The data was obtained from Yahoo Finance and the period spanned from February 21, 2012, until February 23, 2023. Each dataset was divided into in-sample data and post-sample for fitting the model and testing the models' performance respectively. To estimate the parameters of each model, in-sample data from February 21, 2012, and August 31, 2019, was used. The post-sample data, spanning from September 1, 2019, to February 23, 2023, inclusive was used to evaluate the model's performance in the COVID-19 recession.

The daily returns of the six indices are displayed in Figure 1. Table 2 displays some summary statistics for the data. Examining Figure 1 and contrasting the unconditional mean and standard deviation during non-COVID and COVID periods, it becomes evident that stock markets experienced diminished returns and heightened volatility amid the COVID-19 pandemic. A closer look at the skewness and kurtosis of the six stock indexes reveals that the data exhibited near symmetry and tails that were lighter in a normal distribution in non-COVID times. Conversely, during the COVID-19 pandemic, the data displayed high skewness and heavier tails, signalling an increased incidence of extreme losses, particularly in the tumultuous year of 2020. The results displayed in Table 2 indicate that the Jarque-Bera test is not supported for any of the series. This implies that the log returns do not conform to a normal distribution, so the student-t distribution is selected for parameter estimation. Furthermore, the null hypothesis of the ADF test is similarly refuted, indicating the absence of a unit root in the log return series for all the examined series, signifying that they are stationary.

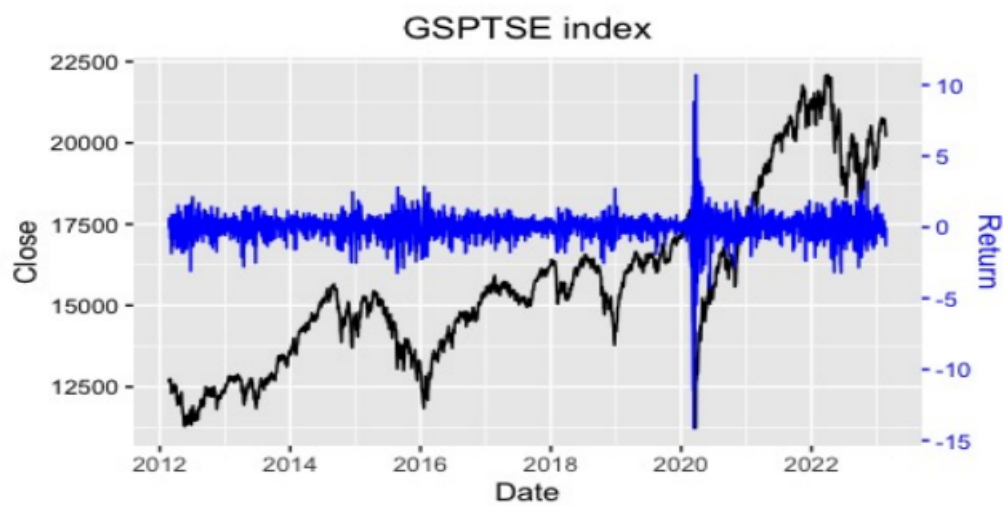
Table 2: Summary statistics

	Mean	Std. dev	Skewness	Kurtosis	ACF	Jarque-Bera
In-sample (non-Covid-19 period)						
GSPTSE	0.012	0.692	-0.437	2.013	-12.771	371.560
SPY	0.038	0.816	-0.464	3.282	-12.587	898.620
FCHI	0.019	1.098	-0.422	3.777	-13.392	1175.500
AEX	0.024	0.965	-0.410	2.877	-13.414	703.530
NIKKEI	0.036	1.317	-0.505	4.435	-12.636	1552.900
HSI	0.010	1.067	-0.382	2.073	-11.372	369.430
Post-sample (Covid-19 period)						
GSPTSE	0.013	1.305	-2.323	35.183	-8.305	48558.000
SPY	0.022	1.511	-1.109	13.618	-8.790	7367.400
FCHI	0.019	1.440	-1.323	14.184	-9.458	8187.200
AEX	0.021	1.321	-1.101	11.084	-9.142	5022.700
NIKKEI	0.020	1.290	-0.035	3.354	-9.856	424.610
HSI	-0.050	1.542	0.167	3.035	-9.452	354.260

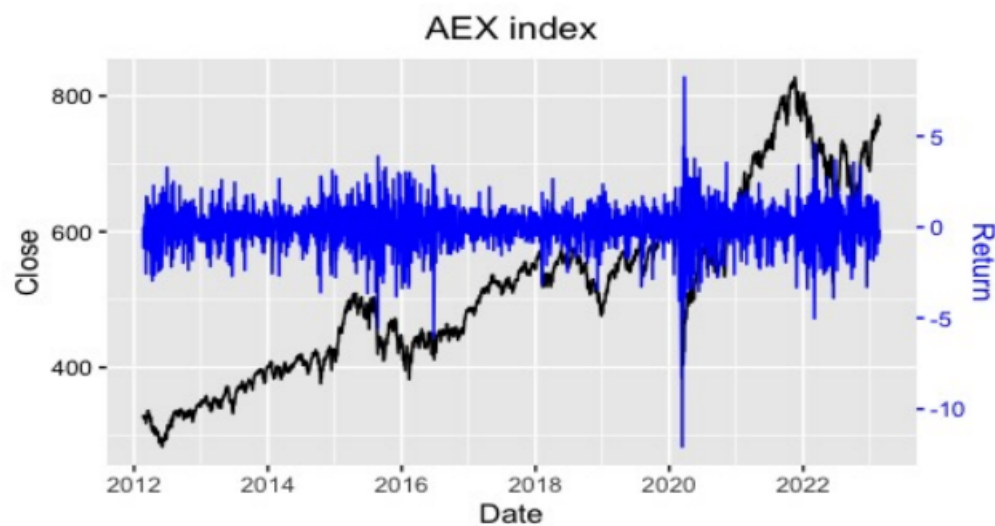
Figure 1: The daily returns of six indices (United States SPY 500 Canada GSPTSE, Hong Kong HSI, Japan NIKKEI, Amsterdam AEX, France FCHI) from February 21, 2012, until February 23, 2023.



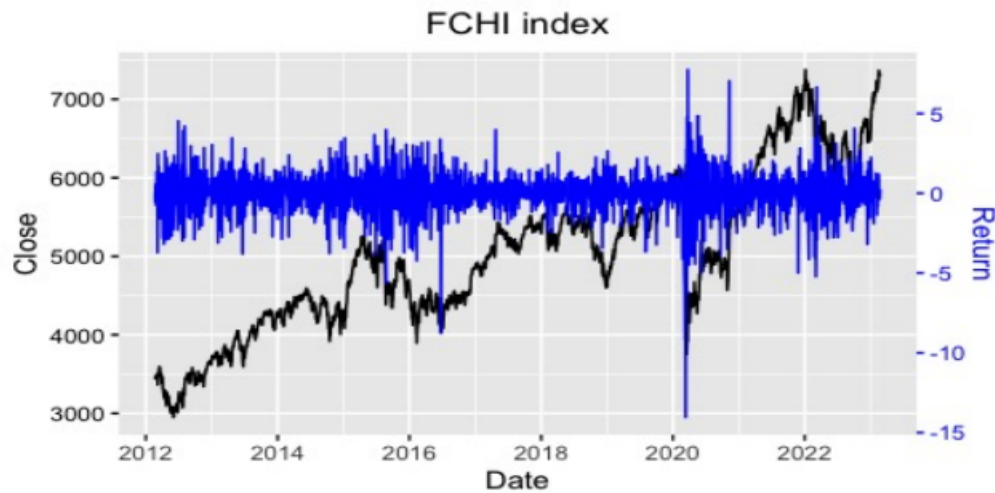
Source: Yahoo Finance



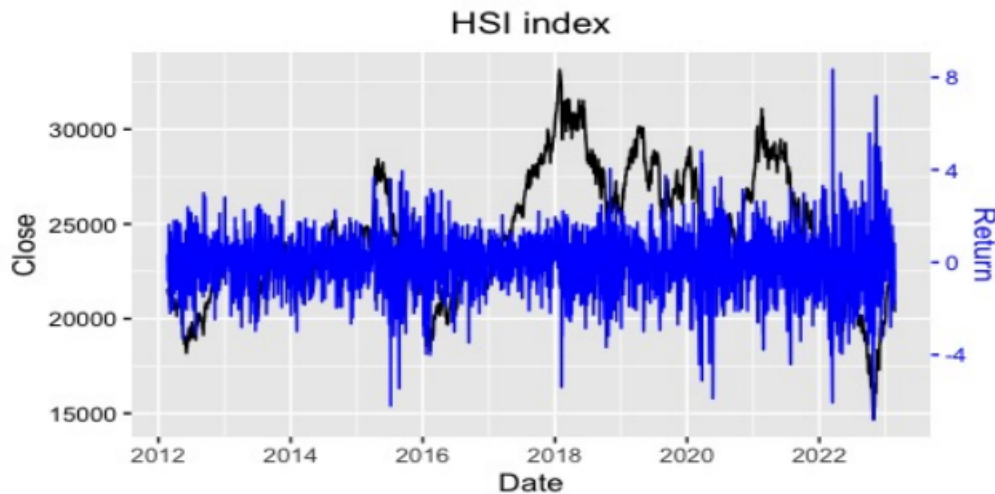
Source: Yahoo Finance



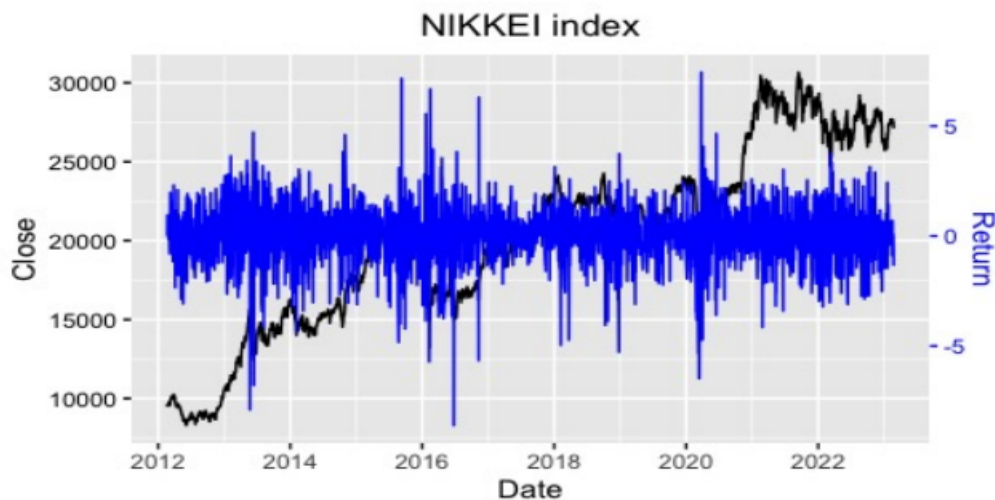
Source: Yahoo Finance



Source: Yahoo Finance



Source: Yahoo Finance



Source: Yahoo Finance

Evaluation Methods

Numerous studies have recently proposed various loss functions and methods of evaluation for VaR model backtesting. However, numerous significant properties of diverse models are

consistent with various evaluation methodologies (González-Rivera, Lee, & Mishra, 2004). Consequently, in this empirical study, the two most commonly used accuracy measurement and assessment methods Kupiec's test and dynamic quantile (DQ) test are used as evaluation indicators. For the following typical risk levels: 1% and 5%, the day-ahead estimation of the VaR was examined in the empirical investigation. A comprehensive back-testing exercise, involving both unconditional and conditional coverage tests, was carried out to evaluate the VaR accuracy in the post-sample period. The likelihood ratio is used to test the risk measurement model, in which the Kupiec test is one of the main unconditional coverage tests to study and evaluate the model's competence to cover risk (Kupiec, 1995). The equivalent equation is written as follows:

$$2\ln[(1 - \frac{N}{T})^{T-N}(\frac{N}{T})^N] - 2\ln[(1 - p)^{T-N}p^N] \sim \chi_1^2$$

The sample's observed number of exceptions is N . T is the total number of observations, and p is the expected frequency of exceptions. This test is under χ_1^2 -distributed with one degree of freedom. A model can be rejected for both high and low failure when this test is applied, however, its power could be poor stated by Kupiec (1995). Therefore, attention is shifted to an alternative criterion. This study employed the DQ test (a joint test of proper unconditional coverage and independence of violations) proposed by Engle and Manganelli (2004) to assess conditional coverage. It determines whether the hit variable's sequence, is defined as $\text{Hit}_t = I(r_t < q_t(\theta)) - \theta$, is independent of the Hit_t variable's lags and the conditional quantile estimates, and whether its distribution is i.i.d. Bernoulli with probability. In the regression test, four lags of the Hit variable were applied, following the methodology of Engle and Manganelli (2004).

Post-Sample Evaluation Results

Results of VaR Risk Measurement Based on the Kupiec Test

As shown in Table 3, under the 95% VaR confidence level, the performance of several models on the six stock indices is shown. In this evaluation, one-step estimation is applied, and the model provides better accurate estimation as reflected by the smaller number of test rejections. Within this calibration process, the majority of conventional, asymmetric, adaptive, and long-memory models prove inadequate in delivering precise VaR forecasting at the 95% confidence level. Only IGARCH and FIGARCH yield acceptable predictions, although with two deviations from the expected outcomes. Additionally, the findings highlight that STES-IndVol performs equivalently to IGARCH and FIGARCH in terms of prediction accuracy. Notably, STES-E outperforms other models by achieving the fewest violations at the 95% confidence level. It's essential to emphasize that the STES approach generally produces less accurate predictions when trade volume is employed as the transition variable for VaR forecasting at a 95% confidence level. Table 4 focuses on the models' abilities to predict VaR at 99% confidence levels for more extreme quantiles. Based on the findings, both the STES-E and FIGARCH models successfully passed the calibration test without any violations, maintaining a 99% confidence level. Furthermore, IGARCH, STES-IndVol, and STES-InVol demonstrate accuracy, with just one prediction failure at the same confidence level. It's noteworthy that incorporating trading volume as a transition variable enhances the STES model's ability to produce acceptable VaR forecasting, particularly at the 99% confidence level. Regarding the selection of transition variables, it is evident that STES models employing a single transition variable tend to yield superior predictions compared to those with multiple transition variables, a trend observed across both confidence levels.

Table 3: 95 % Kupie's Test - LR Value

	GSPTSE	SPY	FCHI	AEX	NIKKEI	HSI	Number significant at 5% level
GARCH	1.372	7.931	2.126	8.006	2.687	7.331	3
IGARCH	1.372	7.189	1.380	4.061	1.127	3.006	2
GJRGARCH	5.271	13.057	3.523	4.634	3.174	13.308	4
EGARCH	9.665	18.161	6.557	9.583	1.828	12.359	5
FIGARCH	0.347	4.554	1.065	4.634	1.828	3.516	2
AFIMA	19.490	58.836	12.182	33.143	4.260	47.730	6
LSTGARCH	3.534	34.652	4.634	10.418	33.008	9.700	5
ESTGARCH	2.556	23.974	3.021	14.066	0.221	10.555	3
STES-E	1.055	8.706	0.553	2.555	0.589	2.533	1
STES-AE	4.078	18.161	8.006	3.523	3.174	9.700	4
STES-SE	4.078	15.010	10.418	14.066	1.828	10.555	5
STES-E&AE	4.078	15.010	5.241	4.061	3.699	9.700	5
STES-E&SE	4.078	15.010	9.583	2.126	1.458	8.878	4
STES-IndVol	1.055	9.514	3.021	9.583	0.221	3.006	2
STES-IndVol&AE	4.657	13.057	9.583	8.778	0.837	10.555	5
STES-IndVol&SE	4.078	13.057	10.418	14.066	0.837	11.441	5
STES-InVol	1.055	11.223	13.109	5.882	0.837	4.063	4
STES-InVol&AE	4.078	14.019	21.566	3.021	3.174	11.441	4
STES-InVol&SE	4.078	15.010	11.285	62.115	7.600	12.359	6

Note: Number of test rejections for the six indices

Table 4: 99 % Kupie's Test - LR Value

	GSPTSE	SPY	FCHI	AEX	NIKKEI	HSI	Number significant at 5% level
GARCH	9.566	2.139	7.662	6.286	0.112	8.417	3
IGARCH	8.043	1.372	3.881	5.024	0.112	4.404	1
GJRGARCH	3.090	1.372	10.730	9.145	0.428	6.964	3
EGARCH	8.043	5.275	9.145	10.730	0.113	6.964	4
FIGARCH	4.144	2.139	5.024	3.881	0.112	3.312	0
AFIMA	14.729	29.614	16.054	12.413	0.477	19.227	5
LSTGARCH	8.043	37.118	9.145	7.662	41.203	4.404	5
ESTGARCH	6.628	27.248	9.145	12.413	0.112	4.404	3
STES-E	5.326	3.052	6.286	5.024	0.113	3.312	0
STES-AE	11.191	7.979	14.189	6.286	7.085	6.964	5
STES-SE	11.191	5.275	16.054	12.413	4.498	6.964	4
STES-E&AE	11.191	9.495	14.189	7.662	8.552	6.964	6
STES-E&SE	11.191	5.275	14.189	5.024	3.392	5.624	2
STES-IndVol	5.326	6.570	9.145	6.286	0.000	3.312	1
STES-IndVol&AE	11.191	7.979	16.054	6.286	0.428	6.964	4
STES-IndVol&SE	11.191	7.979	16.054	12.413	0.428	6.964	5
STES-InVol	5.326	6.570	31.335	6.286	1.597	4.404	1
STES-InVol&AE	11.191	12.829	36.338	7.662	7.085	8.417	6
STES-InVol&SE	11.191	11.113	18.005	127.990	11.803	8.417	6

Note: Number of test rejections for the six indices

Results of VaR Risk Measurement Based on the Dynamic Quantile Test

The p-value for the dynamic quantile test is shown in Tables 5 and 6 at 95% and 99% confidence levels, respectively. For the dynamic quantile test, larger p-values are preferable, and fewer test rejections signify accurate VaR forecasting. The majority of models struggle to generate accurate VaR forecasting, as they are unable to pass more than half of the DQ test at the 95% confidence level. Notably, only the FIGARCH and STES-E models achieve this goal. At the higher 99% confidence level, FIGARCH demonstrates superior performance, followed by GARCH, IGARCH, and the STES-E model. It's crucial to underscore that most models exhibit poor forecasting abilities at both confidence levels, except for the FIGARCH and STES-E models. This diminished prediction accuracy can be attributed to the significant fluctuations experienced during the COVID-19 period. For example, the HSI and SPY indexes witnessed rapid increases in volatility during this time, resulting in the majority of models struggling to deliver precise VaR forecasting.

Table 5: 95 % DQ Test - P Value

	GSPTSE	SPY	FCHI	AEX	NIKKEI	HSI	Number significant at 5% level
GARCH	0.755	0.024	0.063	0.000	0.012	0.004	4
IGARCH	0.533	0.033	0.045	0.002	0.037	0.034	5
GJRGARCH	0.205	0.005	0.291	0.020	0.097	0.001	3
EGARCH	0.008	0.000	0.066	0.009	0.226	0.003	4
FIGARCH	0.552	0.140	0.188	0.008	0.051	0.035	2
AFIMA	0.000	0.230	0.000	0.000	0.000	0.000	5
LSTGARCH	0.222	0.000	0.003	0.003	0.000	0.001	5
ESTGARCH	0.581	0.000	0.250	0.000	0.807	0.000	3
STES-E	0.778	0.026	0.122	0.030	0.268	0.052	2
STES-AE	0.296	0.000	0.001	0.001	0.005	0.000	5
STES-SE	0.299	0.000	0.001	0.000	0.034	0.000	5
STES-E&AE	0.296	0.000	0.012	0.009	0.004	0.000	5
STES-E&SE	0.298	0.001	0.001	0.055	0.112	0.001	3
STES-IndVol	0.764	0.006	0.011	0.000	0.306	0.042	4
STES-IndVol&AE	0.238	0.001	0.001	0.000	0.183	0.000	4
STES-IndVol&SE	0.299	0.001	0.001	0.000	0.183	0.000	4
STES-InVol	0.764	0.001	0.000	0.000	0.110	0.006	4
STES-InVol&AE	0.296	0.000	0.000	0.011	0.006	0.000	5
STES-InVol&SE	0.298	0.000	0.000	0.000	0.000	0.000	5

Note: Number of test rejections for the six indices

Table 6: 99 % DQ Test - P Value

	GSPTSE	SPY	FCHI	AEX	NIKKEI	HSI	Number significant at 5% level
GARCH	0.000	0.339	0.011	0.017	0.881	0.000	2
IGARCH	0.000	0.566	0.085	0.043	0.866	0.000	2
GJRGARCH	0.096	0.516	0.002	0.009	0.365	0.007	3
EGARCH	0.000	0.001	0.009	0.001	0.429	0.006	5
FIGARCH	0.070	0.354	0.039	0.115	0.846	0.000	1
AFIMA	0.000	0.000	0.000	0.000	0.000	0.000	6
LSTGARCH	0.000	0.000	0.006	0.022	0.000	0.088	4
ESTGARCH	0.021	0.000	0.010	0.001	0.652	0.054	3
STES-E	0.000	0.265	0.011	0.038	0.628	0.000	2
STES-AE	0.000	0.008	0.000	0.018	0.003	0.000	5
STES-SE	0.000	0.068	0.000	0.000	0.016	0.000	4
STES-E&AE	0.000	0.002	0.000	0.010	0.001	0.000	5
STES-E&SE	0.002	0.069	0.000	0.057	0.048	0.000	3
STES-IndVol	0.000	0.013	0.000	0.012	0.742	0.000	3
STES-IndVol&AE	0.000	0.009	0.000	0.012	0.610	0.000	4
STES-IndVol&SE	0.000	0.007	0.000	0.000	0.602	0.000	5
STES-InVol	0.000	0.004	0.000	0.000	0.129	0.000	5
STES-InVol&AE	0.000	0.000	0.000	0.011	0.003	0.000	5
STES-InVol&SE	0.000	0.000	0.000	0.000	0.000	0.000	6

Note: Number of test rejections for the six indices

Performance Comparison and Discussion

The Kupies and dynamic quantile test results are shown in Table 7 with 95% and 99% confidence levels. It offers a thorough assessment of the performance of several models at various quantiles. The smaller value suggests that the model makes more accurate predictions. Upon evaluating the performance of all models using both calibration tests, STES-E emerges as the top performer in VaR forecasting, particularly at a 95% confidence level. Furthermore, FIGARCH proves to be an effective model, especially at lower confidence levels. This can be attributed to the presence of numerous unforeseen extreme losses during the COVID-19 pandemic, causing most models to underestimate risk, particularly at lower confidence levels. Focusing on the performance in extreme tail situations, the outcomes reveal that STES-E exhibits reliability with satisfactory test results. Additionally, STES-E stands out as significantly superior to conventional GARCH models, asymmetric models, adaptive models, and some long-memory models, except for the FIGARCH model. Notably, the results indicate that, particularly at high VaR confidence levels, smooth transition models excel in generating precise VaR forecasting. These findings align with recent studies, which underscore the effectiveness of time-varying and smooth transition models in enhancing VaR prediction accuracy, especially at higher VaR confidence levels. (Chen, Gerlach, & Lin, 2014; Guermat & Harris, 2002). Indeed, prior investigations have demonstrated that when it comes to VaR forecasting in the context of the global financial crisis, long memory exerts the most significant influence, followed by the leverage effect, particularly at lower confidence levels (Su, 2014). This explains the exceptional performance of FIGARCH during the COVID-19

pandemic. Due to the inherent robustness of the STES model in forecasting volatility, the STES-E model exhibits superior VaR forecasting capabilities at both confidence levels amid the COVID-19 pandemic, even outperforming the long-memory FIGARCH model at lower confidence levels. Additionally, the evidence indicates that using trading volume as the sole transition variable is deemed acceptable primarily at higher confidence levels. The selection of transition variables presents limitations, as the majority of them fail to enhance the model's forecasting ability. Additionally, the findings suggest that when the STES model integrates a single transition variable rather than multiple transition variables, VaR forecasting tends to be more accurate.

Table 7: Comparison of the number of rejections for Kupiec's test and dynamic quantile tests at the 5% and 1% significance level.

	5% Probability level			1% Probability level		
	Kupie's test	DQ Test	Total	Kupie's test	DQ Test	Total
GARCH	3	4	7	3	2	5
IGARCH	2	5	7	1	2	3
GJRGARCH	4	3	7	3	3	6
EGARCH	5	4	9	4	5	9
FIGARCH	2	2	4	0	1	1
AFIMA	6	5	11	5	6	11
LSTGARCH	5	5	10	5	4	9
ESTGARCH	3	3	6	3	3	6
STES-E	1	2	3	0	2	2
STES-AE	4	5	9	5	5	10
STES-SE	5	5	10	4	4	8
STES-E&AE	5	5	10	6	5	11
STES-E&SE	4	3	7	2	3	5
STES-IndVol	2	4	6	1	3	4
STES-IndVol&AE	5	4	9	4	4	8
STES-IndVol&SE	5	4	9	5	5	10
STES-InVol	4	4	8	1	5	6
STES-InVol&AE	4	5	9	6	5	11
STES-InVol&SE	6	5	11	6	6	12

Note(s): Smaller values are preferred.

Conclusions

This study contributes to the body of research on VaR forecasting in the stock market by applying adaptive models like the STES method. In the investigation of six stock indices, the STES method shows promising results. It performs particularly well at the low confidence level when the size of the shock from the prior period is used as the transition variable.

As it's widely recognized, the COVID-19 pandemic brought about numerous unanticipated extreme losses, leading to the underestimation of risk by most models. Previous studies have shown that during financial crises, long memory had the greatest impact on VaR forecasting, followed by the leverage effect, especially at lower confidence levels (Su, 2014). This

accounts for the exceptional effectiveness of FIGARCH throughout the COVID-19 crisis. Moreover, thanks to the robustness of the STES model in predicting market volatility, STES-E outperforms the accuracy of VaR forecasting at all confidence intervals during the pandemic, even exceeding FIGARCH's performance at lower confidence levels.

In the empirical analysis of the COVID-19 pandemic, this study also explored the choice of transition variables. The results emphasize the importance of using the previous period's shock magnitude for optimal VaR predictions. Employing trading volume alone as a transition variable can yield acceptable VaR forecasting at a high confidence level. Moreover, the findings indicate that simplifying the STES model by utilizing a single transition variable, rather than multiple ones, enhances the accuracy of VaR forecasting.

Based on the results, this study has several practical implications, particularly for Asian financial institutions and regulatory bodies. The findings suggest that adopting the STES model can significantly improve risk forecasting accuracy, particularly at lower confidence levels during periods of market turbulence like the COVID-19 pandemic, which is particularly beneficial for developing stock markets. This increased precision can help financial institutions in Asia better assess potential losses, leading to more effective capital allocation and strategic planning (Kose et al., 2009). As a result, these advancements can contribute to a stronger and more stable financial system in the region. Additionally, the successful application of advanced risk management strategies like STES can help maintain appropriate capital reserve levels, enhancing financial stability in Asian markets.

This study faces three primary constraints. Firstly, it employs solely the student-t distribution for parameter estimation, which may not be enough to evaluate the benchmark VaR model performance since asymmetric distribution-based VaR models might yield improved forecasts. Secondly, a broader range of stock indices should be included in the empirical analysis for a more extensive comparison. Lastly, the performance of these models during periods unaffected by COVID-19 has not been assessed, indicating the potential necessity for additional research to confirm the effectiveness of STES methods in non-pandemic conditions.

In future studies, considering alternative transition variables and models with more than two transition variables may perform better because they may include more information, enabling more accurate and rapid changes in the time-variable parameters. Another potential method for enhancing the performance of the model is to use realized volatility constructed by high-frequency data in parameter optimization. Incorporating the STES approach into interval forecasting is another potential application. Furthermore, by tailoring these models to the unique characteristics such as the risk appetite of Asian markets, future research can offer targeted insights to inform policy development and business strategies, fostering responsible practices and innovation in the region.

References

- Abad, P., & Benito, S. (2013). A detailed comparison of value at risk estimates. *Mathematics and Computers in Simulation*, 94, 258-276. Doi: [10.1016/j.matcom.2012.05.011](https://doi.org/10.1016/j.matcom.2012.05.011)
- Alexander, C. (Ed.). (2009). *Market Risk Analysis*, Boxset. John Wiley & Sons.
- Angelidis, T., Benos, A., & Degiannakis, S. (2004). The use of GARCH models in VaR estimation. *Statistical Methodology*, 1(1-2), 105-128. Doi: [10.1016/j.stamet.2004.08.004](https://doi.org/10.1016/j.stamet.2004.08.004)
- Baillie, R. T., Bollerslev, T., & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 74(1), 3-30.
- Bali, T. G., Mo, H., & Tang, Y. (2008). The role of autoregressive conditional skewness and kurtosis in the estimation of conditional VaR. *Journal of Banking & Finance*, 32(2), 269-282. Doi: [10.1016/j.jbankfin.2007.03.009](https://doi.org/10.1016/j.jbankfin.2007.03.009)
- Black, F. (1976). The pricing of commodity contracts. *Journal of Financial Economics*, 3(1-2), 167-179. Doi: [10.1016/0304-405X\(76\)90024-6](https://doi.org/10.1016/0304-405X(76)90024-6)
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327. Doi: [10.1016/0304-4076\(86\)90063-1](https://doi.org/10.1016/0304-4076(86)90063-1)
- Brooks, C. (1998). Predicting stock index volatility: can market volume help? *Journal of Forecasting*, 17(1), 59-80. [10.1002/\(SICI\)1099-131X\(199801\)17:1<59::AID-FOR676>3.0.CO;2-H](https://doi.org/10.1002/(SICI)1099-131X(199801)17:1<59::AID-FOR676>3.0.CO;2-H)
- Cao, Y. (2022). Extreme risk spillovers across financial markets under different crises. *Economic Modelling*, 116, 106026. Doi: [10.1016/j.econmod.2022.106026](https://doi.org/10.1016/j.econmod.2022.106026)
- Chang, C. L., Jiménez-Martín, J. Á., Maasoumi, E., McAleer, M., & Pérez-Amaral, T. (2019). Choosing expected shortfall over VaR in Basel III using stochastic dominance. *International Review of Economics & Finance*, 60, 95-113.
- Chen, C. W., Gerlach, R., & Lin, E. M. (2014). Bayesian estimation of smoothly mixing time-varying parameter GARCH models. *Computational Statistics & Data Analysis*, 76, 194-209. Doi: [10.1016/j.csda.2013.09.019](https://doi.org/10.1016/j.csda.2013.09.019)
- Choudhry, T. (1995). Integrated-GARCH and non-stationary variances: Evidence from European stock markets during the 1920s and 1930s. *Economics Letters*, 48(1), 55-59. Doi: [10.1016/0165-1765\(94\)00583-N](https://doi.org/10.1016/0165-1765(94)00583-N)
- Donaldson, R. G., & Kamstra, M. J. (2005). Volatility forecasts, trading volume, and the ARCH versus option-implied volatility trade-off. *Journal of Financial Research*, 28(4), 519-538. Doi: [10.1111/j.1475-6803.2005.00137.x](https://doi.org/10.1111/j.1475-6803.2005.00137.x)
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica: Journal of the Econometric Society*, 50(4), 987-1007. Doi: [10.2307/1912773](https://doi.org/10.2307/1912773)
- Engle, R. F., & Bollerslev, T. (1986). Modelling the persistence of conditional variances. *Econometric Reviews*, 5(1), 1-50. Doi: [10.1080/07474938608800095](https://doi.org/10.1080/07474938608800095)
- Engle, R. F., & Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, 22(4), 367-381. Doi: [10.1198/073500104000000370](https://doi.org/10.1198/073500104000000370)
- Engle, R. F., & Patton, A. J. (2001). What good is a volatility model? *Quantitative Finance*, 1(2), 237. Doi: [10.1088/1469-7688/1/2/305](https://doi.org/10.1088/1469-7688/1/2/305)
- Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance*, 48(5), 1779-1801. Doi: [10.1111/j.1540-6261.1993.tb05128.x](https://doi.org/10.1111/j.1540-6261.1993.tb05128.x)
- González-Rivera, G., Lee, T. H., & Mishra, S. (2004). Forecasting volatility: A reality check based on option pricing, utility function, value-at-risk, and predictive likelihood. *International Journal of Forecasting*, 20(4), 629-645. Doi: [10.1016/j.ijforecast.2003.10.003](https://doi.org/10.1016/j.ijforecast.2003.10.003)
- Gooi, L. M., Choo, W. C., Nassir, A. M., & Immng, S. (2018). Volatility Forecasting of Real Estate Stock in Malaysia with Smooth Transition Exponential Smoothing. *International Journal of Economics and Management*, 12, 731-745.
- Granger, C. W., & Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. *Journal of Time Series Analysis*, 1(1), 15-29. Doi: [10.1111/j.1467-9892.1980.tb00297.x](https://doi.org/10.1111/j.1467-9892.1980.tb00297.x)

- Guermat, C., & Harris, R. D. (2002). Forecasting value at risk allowing for time variation in the variance and kurtosis of portfolio returns. *International Journal of Forecasting*, 18(3), 409-419. Doi: [10.1016/S0169-2070\(01\)00122-4](https://doi.org/10.1016/S0169-2070(01)00122-4)
- Hagerud, G. E. (1997). *A New Non-Linear GARCH Model*. EFI, Economic Research Institute, Stockholm. <https://ex.hhs.se/dissertations/221859-FULLTEXT01.pdf>
- Hoga, Y. (2021). The uncertainty in extreme risk forecasts from covariate-augmented volatility models. *International Journal of Forecasting*, 37(2), 675-686.
- Jiménez-Martín, J. Á., McAleer, M., & Pérez-Amaral, T. (2009). The ten commandments for managing value at risk under the Basel II accord. *Journal of Economic Surveys*, 23(5), 850-855.
- Kin, W. C., Chong, C. W., Nassir, A. M., Habibullah, M. S., & Yusop, Z. (2021). Volatility forecasting performance of Smooth Transition Exponential Smoothing method: Evidence from mutual fund indices in Malaysia. *Asian Economic and Financial Review*, 11(10), 829-859. <http://www.aessweb.com/>
- Kose, M. A., Prasad, E. S., Rogoff, K., & Wei, S. J. (2009). Financial Globalization: A Reappraisal. *IMF Staff Papers*, 56(1), 8-62.
- Kupiec, P. H. (1995). *Techniques for verifying the accuracy of risk measurement models*. Division of Research and Statistics, Division of Monetary Affairs, Federal Reserve Board. https://web.archive.org/web/20170809061153id/http://www.systemic-risk-hub.org/papers/bibliography/Kupiec_1995_Preview.pdf
- Lamoureux, C. G., & Lastrapes, W. D. (1990). Heteroskedasticity in stock return data: Volume versus GARCH effects. *The Journal of Finance*, 45(1), 221-229. Doi: [10.1111/j.1540-6261.1990.tb05088.x](https://doi.org/10.1111/j.1540-6261.1990.tb05088.x)
- Liu, M., Taylor, J. W., & Choo, W. C. (2020). Further empirical evidence on the forecasting of volatility with smooth transition exponential smoothing. *Economic Modelling*, 93, 651-659. Doi: [10.1016/j.econmod.2020.02.021](https://doi.org/10.1016/j.econmod.2020.02.021)
- Longerstaey, J., & Spencer, M. (1996). Estimation and forecast. In Riskmetricstm—technical document (pp. 51-54). Morgan Guaranty Trust Company of New York: New York. <https://my.liuc.it/MatSup/2006/F85555/rmtd.pdf>
- McNeil, A. J., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3-4), 271-300. Doi: [10.1016/S0927-5398\(00\)00012-8](https://doi.org/10.1016/S0927-5398(00)00012-8)
- Meng, X., & Taylor, J. W. (2020). Estimating value-at-risk and expected shortfall using the intraday low and range data. *European Journal of Operational Research*, 280(1), 191-202. Doi: [10.1016/j.ejor.2019.07.011](https://doi.org/10.1016/j.ejor.2019.07.011)
- Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society*, 59(2), 347-370. Doi: [10.2307/2938260](https://doi.org/10.2307/2938260)
- Polanski, A., & Stoja, E. (2010). Incorporating higher moments into value-at-risk forecasting. *Journal of Forecasting*, 29(6), 523-535. Doi: [10.1002/for.1155](https://doi.org/10.1002/for.1155)
- Su, J. B. (2014). Empirical analysis of long memory, leverage, and distribution effects for stock market risk estimates. *The North American Journal of Economics and Finance*, 30, 1-39. Doi: [10.1016/j.najef.2014.07.003](https://doi.org/10.1016/j.najef.2014.07.003)
- Taylor, J. W. (2004). Volatility forecasting with smooth transition exponential smoothing. *International Journal of Forecasting*, 20(2), 273-286. Doi: [10.1016/j.ijforecast.2003.09.010](https://doi.org/10.1016/j.ijforecast.2003.09.010)
- Ung, S. N., Choo, W. C., Sambasivan, M., & Nassir, A. M. (2014). The persistency of international diversification benefits: The role of the asymmetry volatility model. *Asian Academy of Management Journal of Accounting and Finance*, 10(1), 151-165. https://eprints.ncl.ac.uk/file_store/production/271628/79E8056C-0969-4FFC-9A87-AFF05831D7B6.pdf
- Zhang, Y., Choo, W. C., Ho, J. S., & Wan, C. K. (2022). Single or combine? Tourism demand volatility forecasting with exponential weighting and smooth transition combining methods. *Computation*, 10(8), 137. Doi: [10.3390/computation10080137](https://doi.org/10.3390/computation10080137)

